Matrix operations

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{pmatrix}$$ ← This a 3×2 matrix

#rows   #cols.

You can refer to elements of the matrix by using subscripts.

With the example above,

$$A_{12} = 2 \quad , \quad A_{31} = 3$$

1st row, 2nd column.

Addition & Subtraction ← just what you would think.

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 4 \\ 0 & 5 & 1 \end{pmatrix}$$

add corresponding entries

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 0 \end{pmatrix} - \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & 2 \\ -2 & 3 & -1 \end{pmatrix}$$

Subtract corresponding entries

Scalar Multiplication

$$-5\begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 6 \\ 10 & 100 & 1000 \end{pmatrix} = \begin{pmatrix} -10 & -5 & -5 \\ 0 & -15 & -30 \\ -50 & -500 & -5000 \end{pmatrix}$$

multiply all entries by the scalar ← real or complex #.

Matrix Multiplication

If $\underset{n\times k}{A}$ & $\underset{r\times s}{B}$ are matrices, we can only multiply them when the middle dimensions agree $(k=r)$.

Result: $n\times s$ matrix

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 8 \end{pmatrix}\begin{pmatrix} 3 & 0 & 4 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 2 & 1 & -2 \end{pmatrix}$$

$2\times 3 \quad 3\times 4$

The result is a 2×4 matrix

The answer is C, and C_{jk} = dot product of jth row with k^{th} column

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 8 \end{pmatrix} \begin{pmatrix} 3 & 0 & 4 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 2 & 1 & -2 \end{pmatrix}$$

$2 \times \boxed{3}$ $\quad 3 \times 4$

$$= \begin{pmatrix} 1\cdot 3 + 0\cdot 3 + (-1)(1) & 1\cdot 0 + 0\cdot 1 + (-1)(2) & 1\cdot 4 + 0\cdot 0 + (-1)1 & 1\cdot 0 + 0\cdot 0 + (-1)(-2) \\ 2\cdot 3 + 1\cdot 3 + 8\cdot 1 & 0\cdot 2 + 1\cdot 1 + 8\cdot 2 & 2\cdot 4 + 1\cdot 0 + 8\cdot 1 & 0\cdot 2 + 0\cdot 1 + 8(-2) \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 & 3 & 2 \\ 17 & 17 & 16 & -16 \end{pmatrix}$$

$\boxed{\text{Ex}}$

1. $\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 3 & 2 \end{pmatrix}$

2. $\underset{2\times 3}{\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}} \underset{2\times 2}{\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}} =$ Aaahh~ Nooo~

3. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 3 & -1 \end{pmatrix}$

4. $\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$

Matrix Multiplication is <u>not</u> commutative.

The Derivative matrix:

example: $F: \mathbb{R}^3 \to \mathbb{R}$

domain — codomain / target space

$F(x,y,z) = \#.$

Deriv matrix F' is a 1×3 matrix

$$F' = \nabla F = \begin{pmatrix} F_x & F_y & F_z \end{pmatrix}$$

eg. If $F(x,y,z) = xy + y - 2z^2x$

$$F' = \nabla F = (y - 2z^2, x+1, -4zx)$$

$$= \begin{pmatrix} y-2z^2 & x+1 & -4zx \end{pmatrix}$$

$$\nabla F(\underset{x}{1}, \underset{y}{-1}, \underset{z}{2}) = \begin{pmatrix} -1-2(2)^2 & 1+1 & -4(2)(1) \end{pmatrix}$$

$$= \begin{pmatrix} -9 & 2 & -8 \end{pmatrix}$$

In general, if $G: \mathbb{R}^n \to \mathbb{R}^k$, then G' is a $k \times n$ matrix.

Think: $$G(x_1, x_2, \dots, x_n) = \begin{pmatrix} G_1(x_1, x_2, \dots) \\ G_2(x_1, x_2 \dots x_n) \\ \vdots \\ G_k(x_1, x_2, \dots, x_n) \end{pmatrix}$$

Then $$G' = \begin{pmatrix} - \nabla G_1 - \\ - \nabla G_2 - \\ \vdots \\ - \nabla G_k - \end{pmatrix}$$

$$G' = \begin{pmatrix} \frac{\partial G_1}{\partial x_1} & \frac{\partial G_1}{\partial x_2} & \cdots & \frac{\partial G_1}{\partial x_n} \\ \frac{\partial G_2}{\partial x_1} & \frac{\partial G_2}{\partial x_2} & \cdots & \frac{\partial G_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial G_k}{\partial x_1} & \frac{\partial G_k}{\partial x_2} & \cdots & \frac{\partial G_k}{\partial x_n} \end{pmatrix}$$

$k \times n$ matrix.

<u>Example</u> $f : \mathbb{R}^3 \to \mathbb{R}^2$

Let $f(x,y,z) = \begin{pmatrix} xy - \sin(x+2z) \\ xyz + y \end{pmatrix}$

$$\Rightarrow f' = \begin{pmatrix} y - \cos(x+2z) & x & -2\cos(x+2z) \\ yz & xz+1 & xy \end{pmatrix}$$

One interesting property of matrix multiplication:

If A & B are $n \times n$ matrices

then $\det(AB) = \det(A)\det(B)$.